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Prime Factorization

Skill 24: The student will find the prime factorization of a composite number using a factor tree.

Instructional Preparation

Duplicate the following (one per student unless otherwise indicated):

- "Climbing Down the Factor Tree" reference sheet
- "Primed for Factoring" worksheet

Prepare an overhead transparency of each of the following:

- "Climbing Down the Factor Tree" reference sheet
- "Primed for Factoring" worksheet

Recall

Before beginning the **Review** component, facilitate a discussion based on the following questions:

- What is a prime number? (*A whole number larger than one with only two factors*)
- What is a composite number? (*A whole number larger than one with more than two factors*)
- Is 60 prime or composite? (*Composite*)
- What are the factors of 60? (*1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, and 60*)
- Are any of the factors primes? (*Yes, 2, 3 and 5*)

Review

1. Explain to the students that the product $2 \times 2 \times 3 \times 5$ is the prime factorization of 60. Any composite number can be written as a product of prime numbers, and today they will see how to do that using a factor tree.

2. Distribute copies of the "Climbing Down the Factor Tree" reference sheet and display its transparency. Briefly review the terms in the "Words to Know" section located at the bottom of the reference sheet. Tell them to reference these terms as needed during the lesson.

3. Direct attention to the "How to Factor a Number" section and have a volunteer read the bulleted statements. Work through the example on the right, showing how composite numbers are broken down and prime factors are just carried along to the next step. Point out that exponents are used to write the repeated factors in the final answer. Explain that it doesn't matter how the original number is factored; they will always get the same factorization once the number is completely factored. Demonstrate this by creating alternate factor trees for 60 on another transparency, using 3×20, 4×15, and 5×12.

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Prime Factorization *(cont.)*

4. Explain that it might not be easy to immediately identify two factors for a number. It helps to know what numbers are divisible by 2, 3, 5, and other primes. Direct attention to the "Divisibility Tests" section. Have volunteers read the tests for 2, 3, 5, and 11. Explain that they can tell if a number is divisible by 2 or by 5 by looking at the digit in the ones place. This is because 2 and 5 are the prime factors of 10. For divisibility by 3, add up all the digits in the number. If this sum is divisible by 3, then so is the original number. For 11, two sums are needed. Add up the all the digits in the odd places (first, third, fifth, etc.) and separately add up all the digits in the even places (second, fourth, sixth, etc.). Subtract the smaller sum from the larger, and if the difference is divisible by 11, so is the original number. Remind the students that 0 is divisible by any number (except 0).

   Direct attention to the table on the right of this section. Work the divisibility tests for 75. Write the responses on the transparency and have the students write them on their sheet as you ask the following questions.

   - Look at the ones digit in 75; is the digit a 0, 2, 4, 6, or 8? (*No*)
   - Is 75 divisible by 2? (*No*)
   - What is the sum of the digits in 75? (*12*)
   - Is this sum divisible by 3? What does that say about 75? (*Yes; it says that 75 is divisible by 3.*)
   - Is 75 divisible by 5? Why or why not? (*Yes, because the ones digit is a 5.*)
   - What are the odd-numbered digits, and what is their sum? (*The only odd-numbered digit is the first, 7.*)
   - What are the even-numbered digits, and what is their sum? (*The only even-numbered digit is the second, 5.*)
   - What is the difference between the sums; is 75 divisible by 11? (*2; no, since 2 is not divisible by 11.*)

   Work the divisibility tests for 484 in a similar fashion. It is divisible by 2, not divisible by 3 or 5, and divisible by 11.

   Explain that the divisibility tests for 7 and primes larger than 11 are more complicated. If the number is not divisible by a smaller prime, they should keep dividing the original number by larger primes until they get a remainder of 0. Answer any questions about factoring and divisibility tests.

Prime Factorization *(cont.)*

Direct attention to the four problems in the center of the sheet. Explain to the students that they are to work in pairs to create factor trees and find the prime factorizations of the numbers. Encourage them to use the divisibility tests, if needed. Remind them to write their final answers in exponential form. Allow adequate time for task completion. Have volunteers write their solutions on the transparency, and verify their answers ($72 = 2^3 \times 3^2$, $297 = 3^3 \times 11$, $390 = 2 \times 3 \times 5 \times 13$, $125 = 5^3$). Answer any questions about the lesson.

5. Distribute copies of the "Primed for Factoring" worksheet and display its transparency. Have a volunteer read the directions at the top of the worksheet. Have the students work individually to complete the worksheet. When they have completed the worksheet, ask for volunteers to write their answers on the transparency.

Wrap-up

To conclude this lesson, have the students write a complete-sentence response to the following prompt in their math journal or on a sheet of notebook paper. Allow adequate time for task completion and then ask various students to share their responses with the class.

- At the beginning of the lesson, the factors of 60 were listed as: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, and 60, yet the prime factorization of 60 is $2^2 \times 3 \times 5$. What happened to the other factors, like 4, 15, and 30? (*Accept all reasonable answers that reflect that the other factors, aside from 1, are composite numbers that can be broken down into the prime factors.*)

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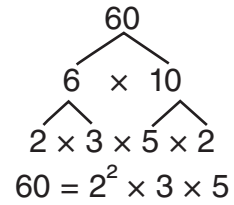
Prime Factorization *(cont.)*

Climbing Down The Factor Tree

How to Factor a Number:

- Write the number as a product of two factors
- Write each factor as the product of two factors
- Continue until all that remains are prime numbers
- Collect repeated factors and write with exponents



$60 = 2^2 \times 3 \times 5$

Divisibility Tests:

2: Ones digit is 0, 2, 4, 6, or 8

3: Sum of the digits is divisible by 3

5: Ones digit is 0 or 5

11: Difference between the sum of the digits in the odd places and the sum of the digits in the even places is divisible by 11

Divisible by:	75	484
2		
3		
5		
11		

1. 72

72 =

2. 297

297 =

3. 390

390 =

4. 125

125 =

Words to Know

Composite number—A whole number larger than 1 that has more than two factors — *4, 6, 8, 9 . . .*

Factor—A number that can be divided into another without remainder — *6 is a factor number of 30*

Prime number—A whole number larger than 1 that has only 1 and itself as factors — *2, 3, 5, 7 . . .*